The Shared Area method

Like convolution, but <u>different</u>

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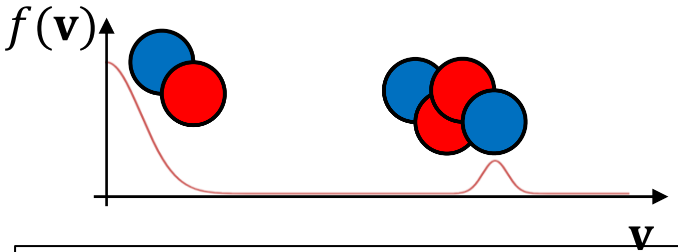
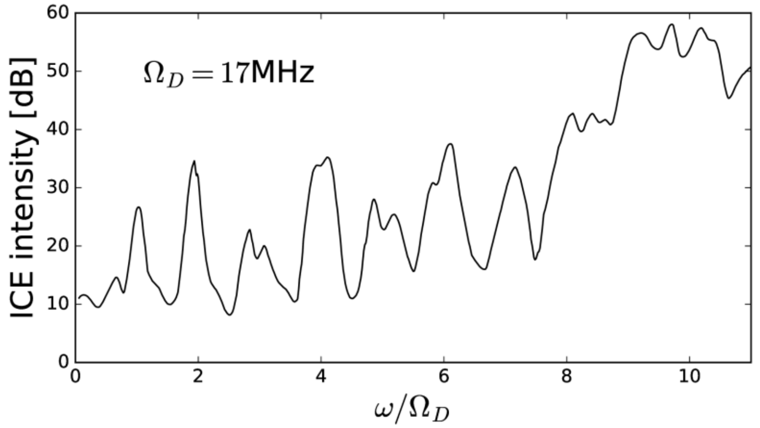
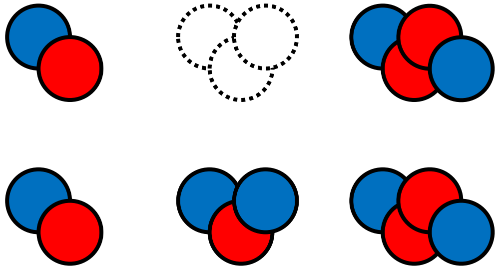
Motivation



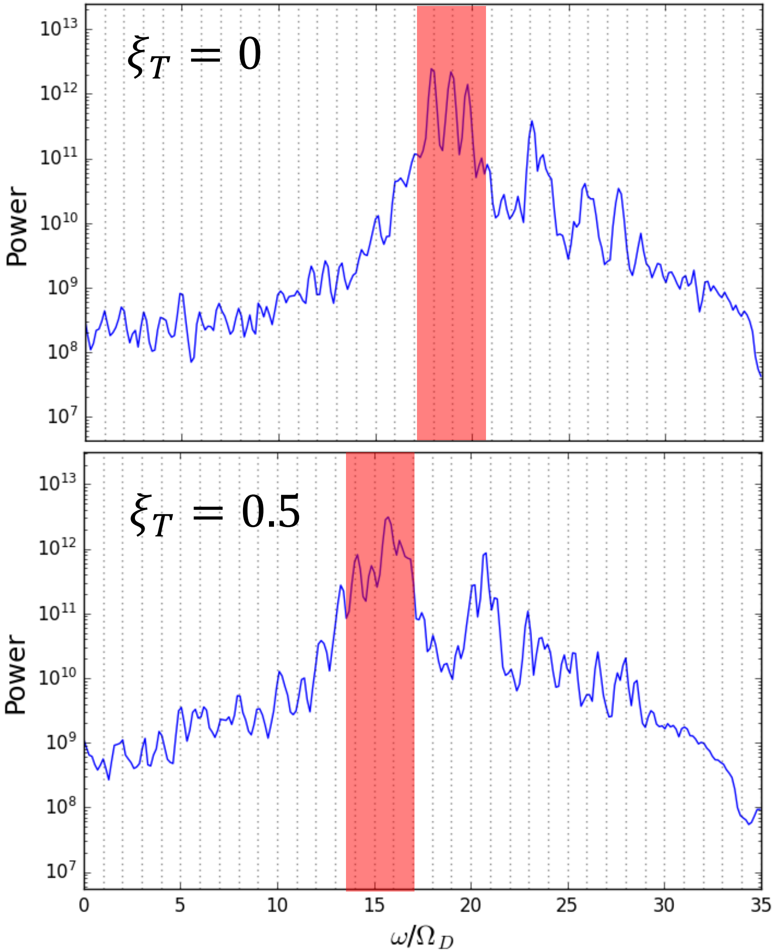
$\partial f/\partial v > 0$ gradients in fusion plasma distribution leads to instability (MCI)



Ion cyclotron emission (ICE) at integer harmonics of energetic minority ($n\Omega_\alpha$)*



Inclusion of tritium in DT fusion plasmas



Power spectral features shift with tritium concentration (ξ_T)

*Adapted from G. A. Cottrell *et al.*, *Nuclear Fusion*, vol. 33, pp. 1365–1387, Sept. 1993

Theory

Convolution

Curve 1

$$\mathrm{Conv}(x) = \int_X f(x) g(x - \Delta x)\ d(\Delta x)$$

Curve 2
(flipped+offset)

- Simple
- Faster (Alternatives / X-Corr)
- More widely known

Shared area

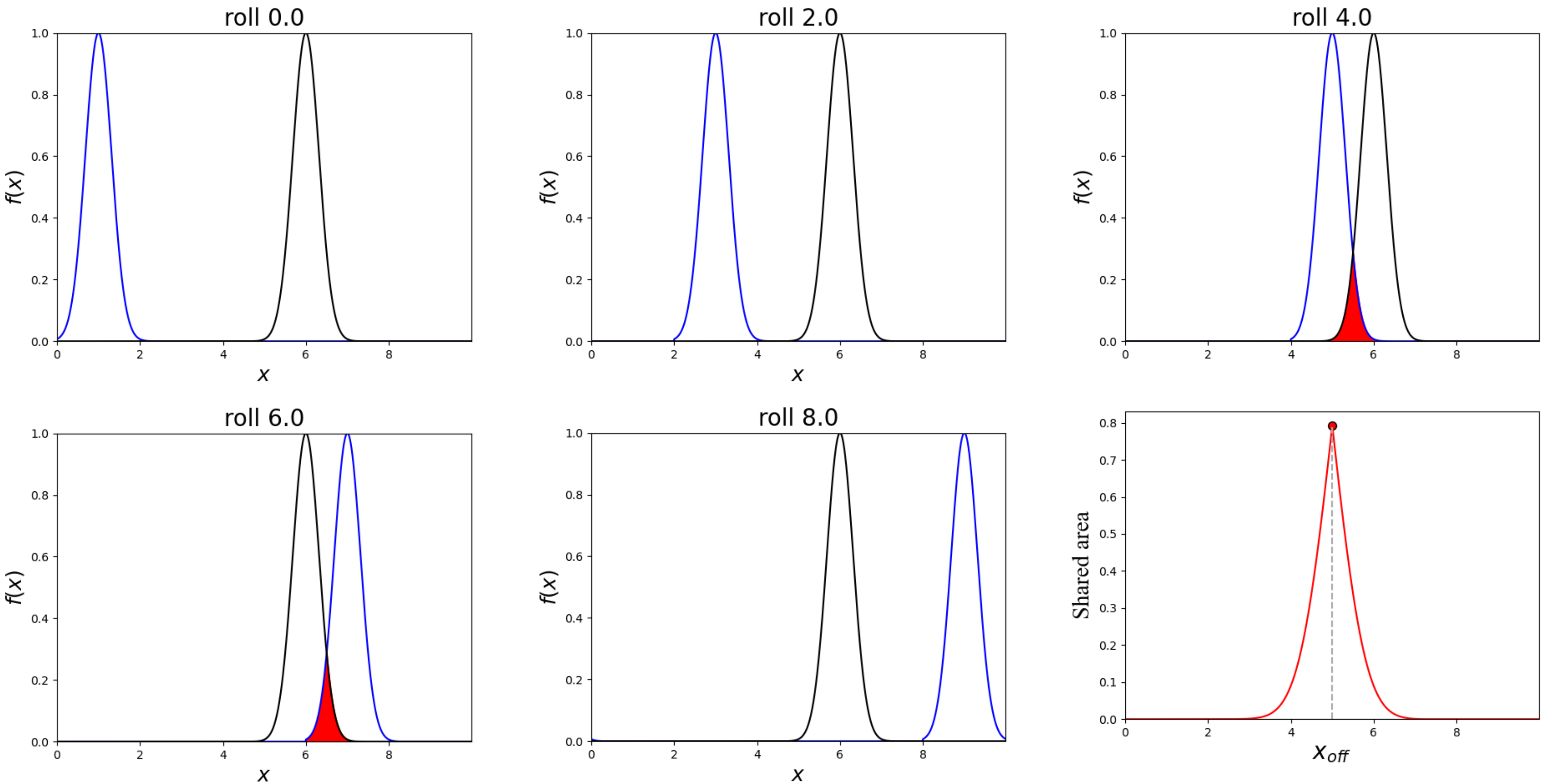
Boolean Boolean

$$\mathrm{SA}(\Delta x) = \int_X \mathrm{d}x\ [B_f(\Delta x, x) f(x) + B_g(\Delta x, x) g(x - \Delta x)]$$

Curve 1 Curve 2
(+offset)

- Reliable against noise (Applications / X-Offsets)
- Novel
- N-dimensional (Applications / Higher dimensions)

Method



Loop over Δx:

    if $f(x) < g(x - \Delta x)$:

        $B_f(\Delta x, x) = 1$

        $B_g(\Delta x, x) = 0$

    else:

        $B_f(\Delta x, x) = 0$

        $B_g(\Delta x, x) = 1$

Applications

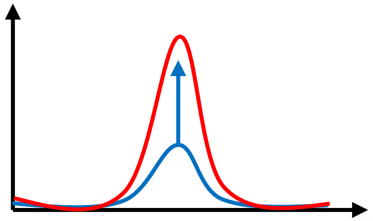
SCALE

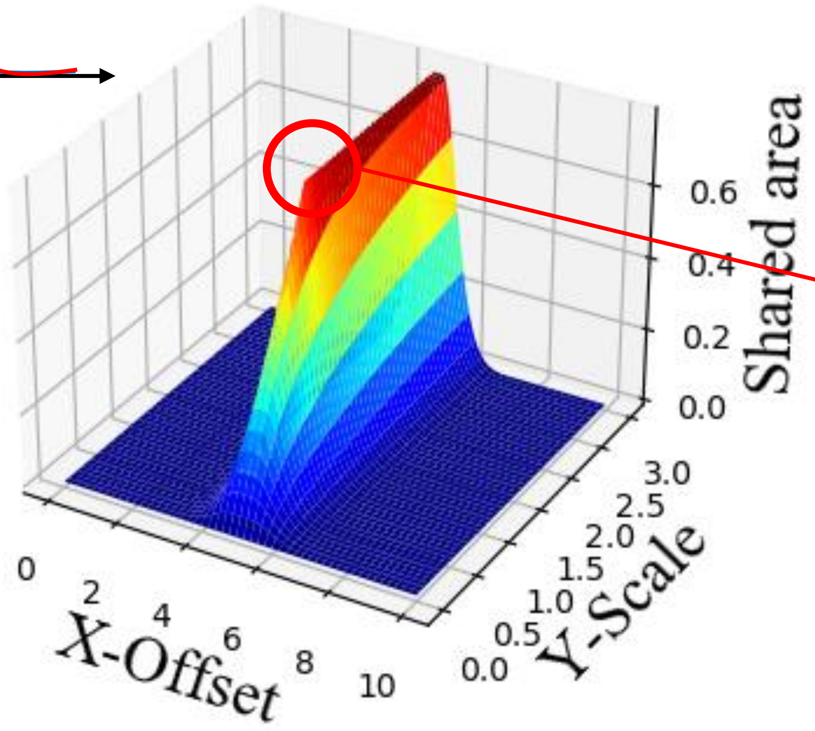
Offsets (↥Y , ↦X)

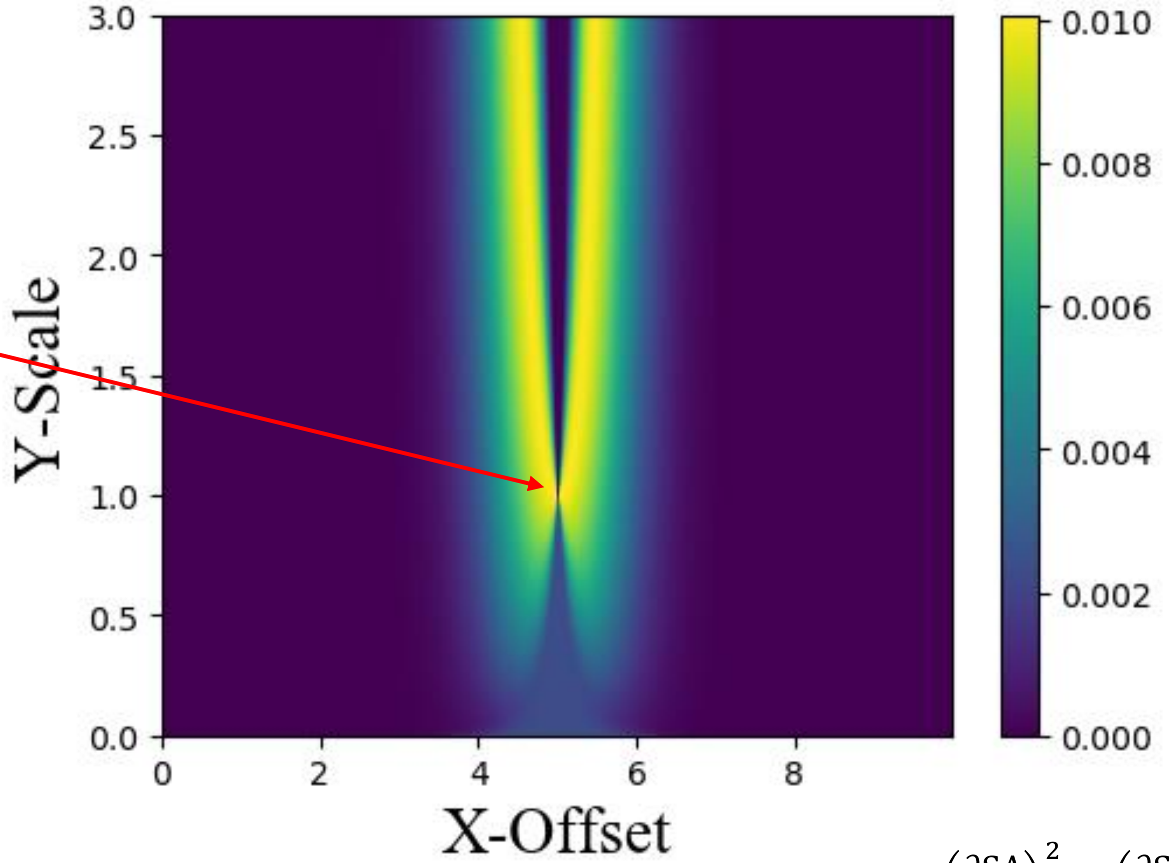
Smoothing

Higher dimensions

Applications / Scale



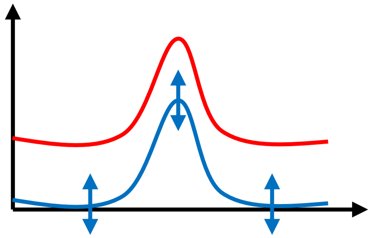


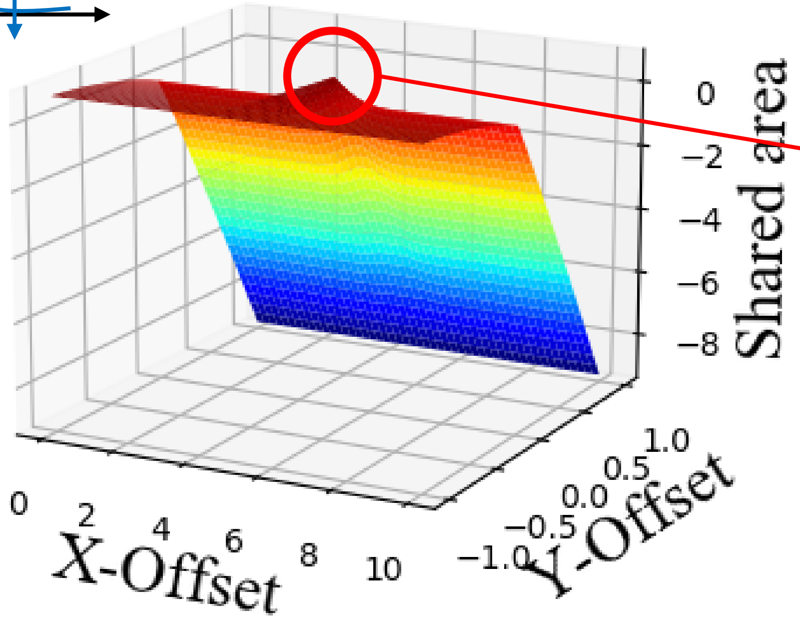


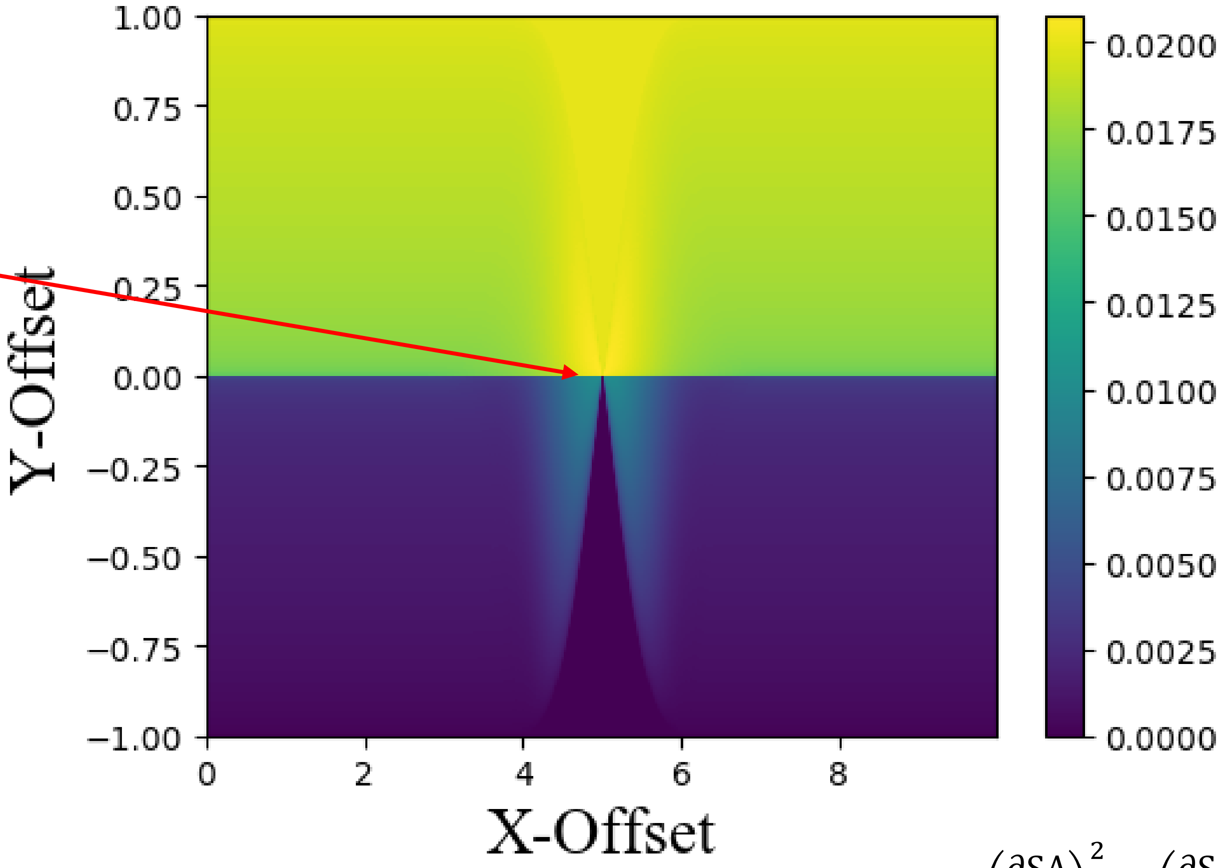
$$\left(\frac{\partial SA}{\partial r}\right)$$

$$\left(\frac{\partial SA}{\partial r}\right)^2 = \left(\frac{\partial SA}{\partial x}\right)^2 + \left(\frac{\partial SA}{\partial y}\right)^2$$

Applications / Y-Offsets







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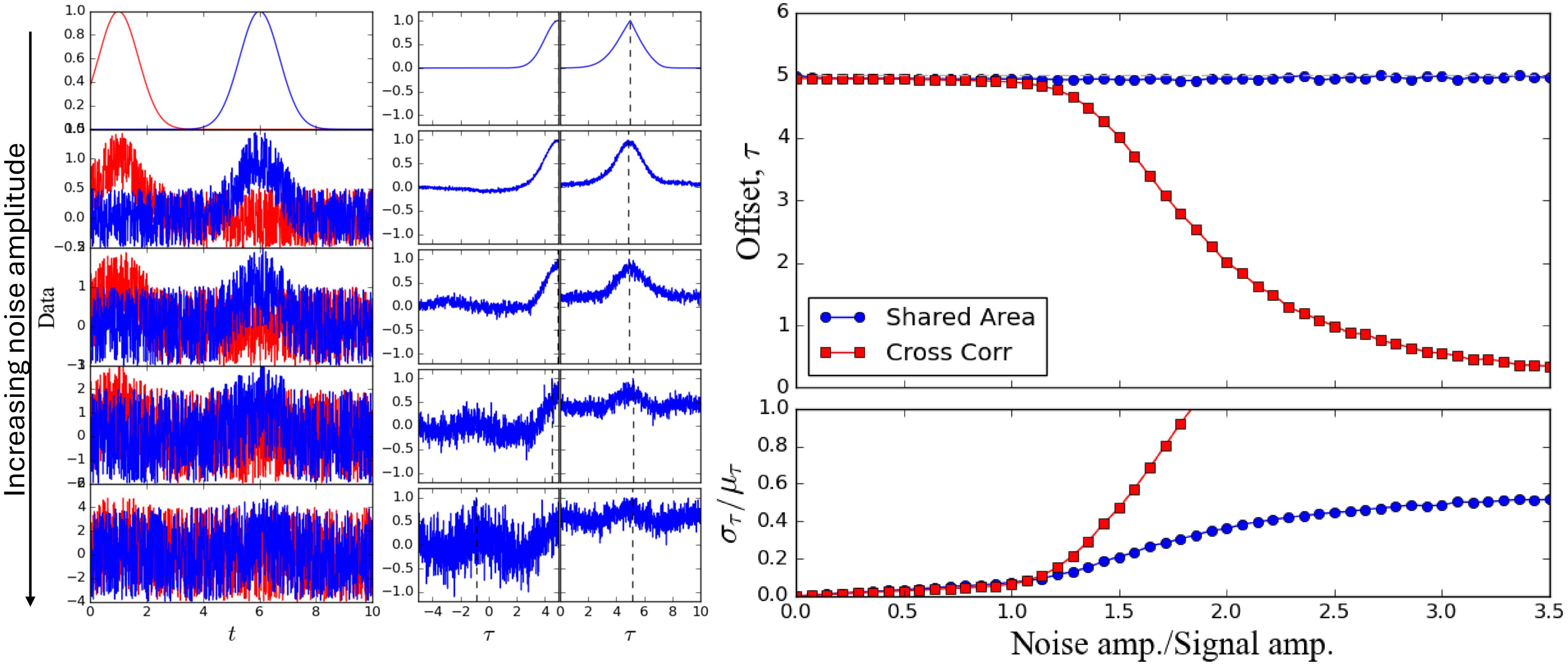
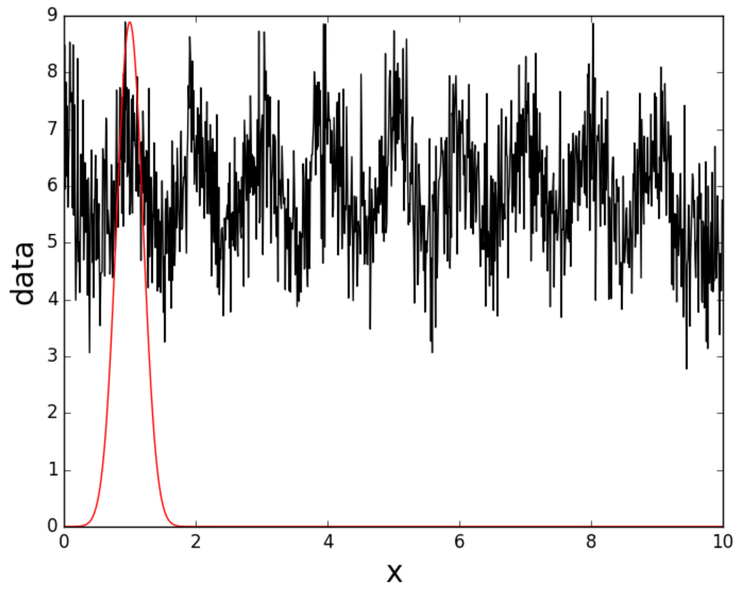
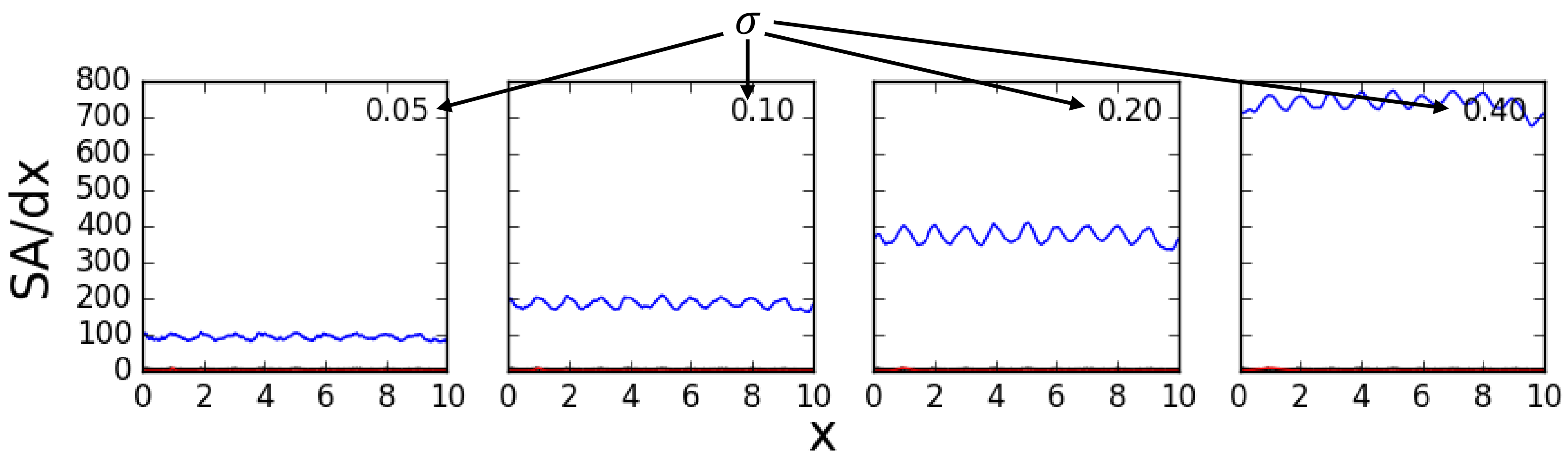
Applications / X-Offsets

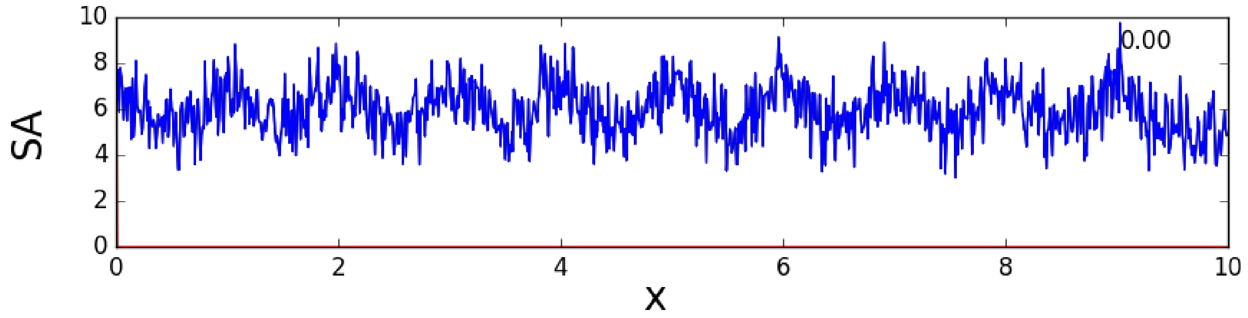


Applications / Smoothing

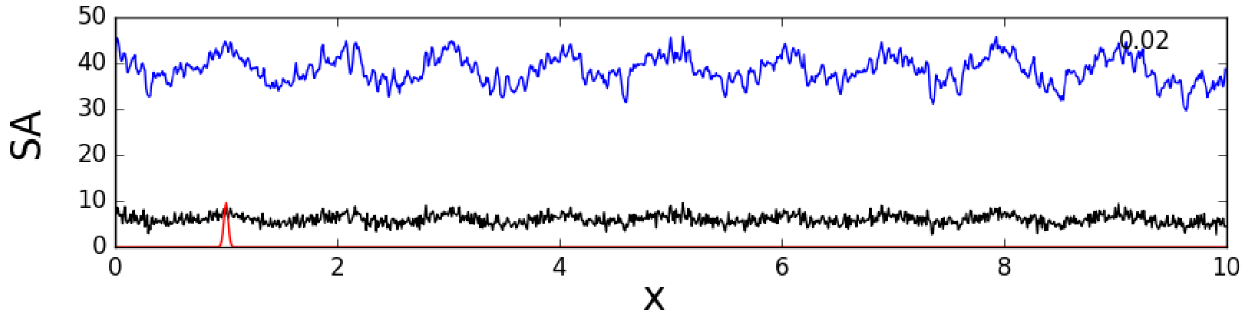




Dirac Delta smoothing returns original for SA/dx



Normalisation by dx no longer valid for Gaussian smoothers



Applications / Higher dimensions

1D :

$$\mathrm{SA}(\Delta x) = \int_X \mathrm{d}x \, [B_f(\Delta x, x) f(x) + B_g(\Delta x, x) g(x - \Delta x)]$$

2D :

$$\mathrm{SA}\,(\Delta x, \Delta y) = \int_X \int_Y \mathrm{d}x \, \mathrm{d}y \left[B_f(\Delta x, \Delta y, x, y) f(x, y) + B_g(\Delta x, \Delta y, x, y) g(x - \Delta x, y - \Delta y)\right]$$

ND :

$$\mathrm{SA}(\boldsymbol{\Delta}) = \int_{\mathcal{R}^N} \mathrm{d}^N \boldsymbol{r} [B_f(\boldsymbol{\Delta}, \boldsymbol{r}) f(\boldsymbol{r}) + B_g(\boldsymbol{\Delta}, \boldsymbol{r}) g(\boldsymbol{r} - \boldsymbol{\Delta})]$$

Offset vector

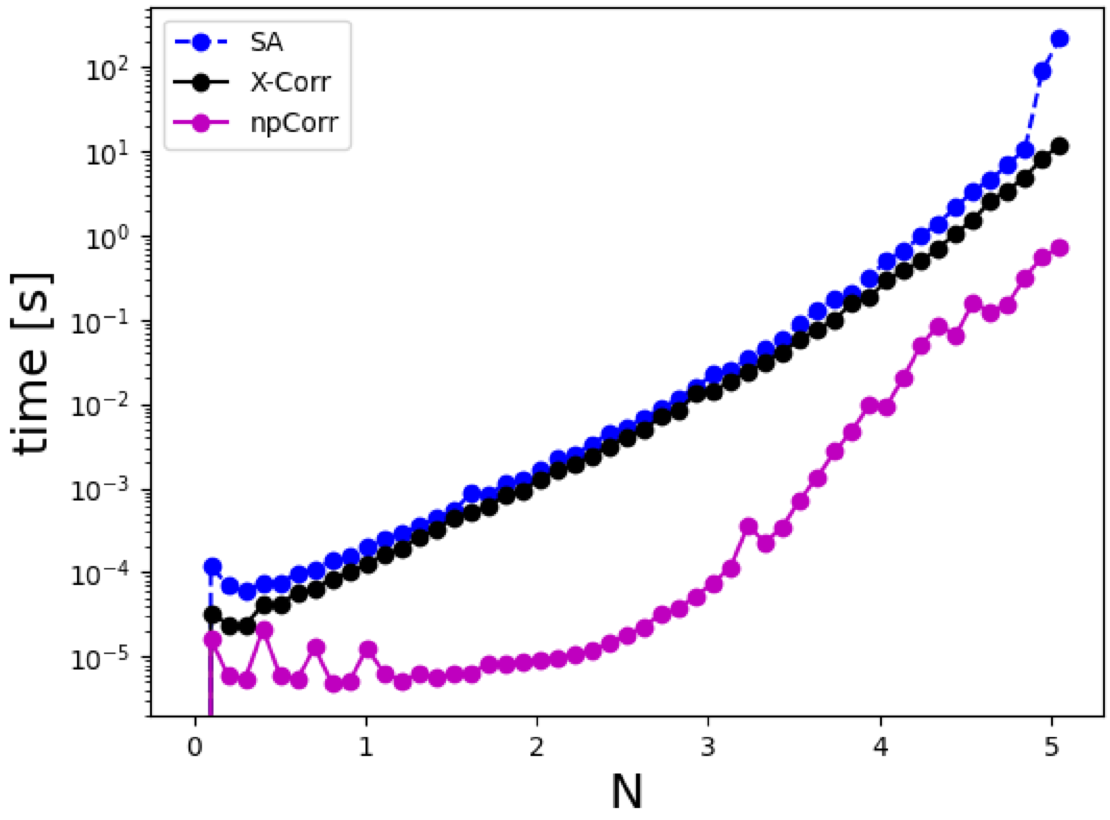
Position vector

Alternatives

1D Cross correlation (X-Corr)

2D Phase correlation (PC)

Alternatives / X-Corr



- Two curves (l_1 and l_2) with random noise
- Number of data points 10^N
- Three methods
  - > Shared-area (SA)
  - > Cross-correlation (X-Corr)
  - > NumPy X-Corr (npCorr)
- NumPy optimised, performs better across all data lengths
- SA falls short throughout, noticeable at extremely large datasets

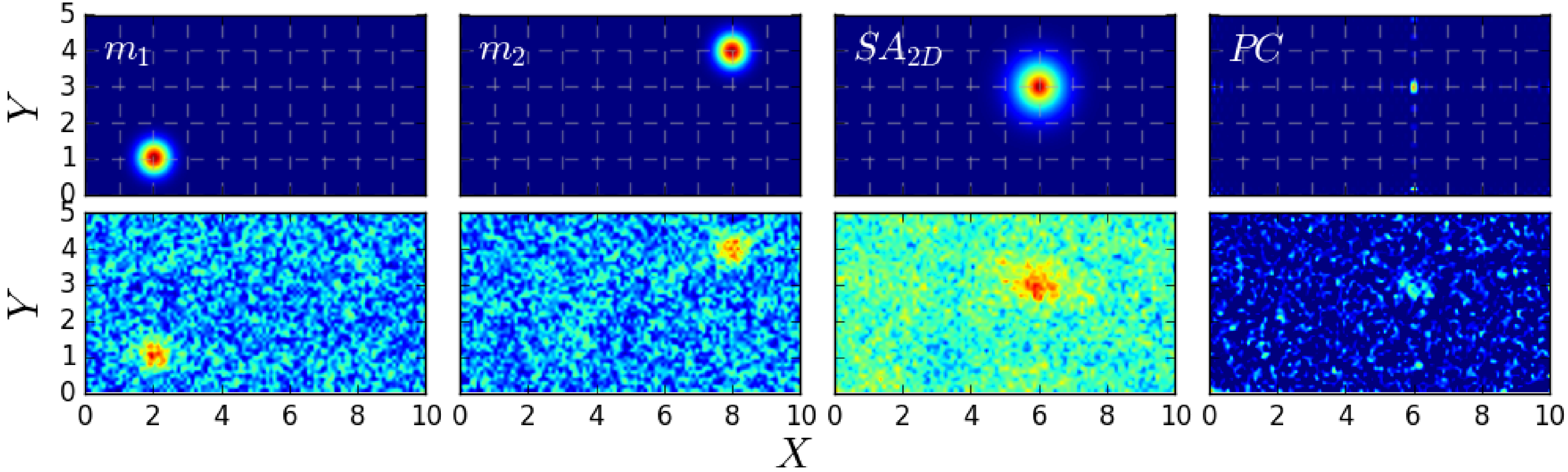
Alternatives / Phase correlation (PC)

Phase correlation: Used to measure translation offsets between two similar datasets

$m_2(x, y) = m_1(x - \Delta x, y - \Delta y)$ — **Assume images same but shifted**

$\mathcal{F}[m_2(a, b)] = \mathcal{F}[\mathrm{m}_1(\mathrm{a}, \mathrm{b})] \cdot \exp\left[-2\pi i \left(\frac{a\Delta x}{N_x} + \frac{b\Delta y}{N_y}\right)\right]$ — **FT would give same, but with phase offset**

$r = \mathcal{F}^{-1}\left[\frac{\mathcal{F}(m_1) \odot \mathcal{F}^*(m_2)}{|\mathcal{F}(m_1) \odot \mathcal{F}^*(m_2)|}\right] = \delta(x + \Delta x, y + \Delta y)$ — **Normalise, take inverse FT, gives Dirac delta at ($\boldsymbol{\Delta x}, \boldsymbol{\Delta y}$)**



No noise

With noise

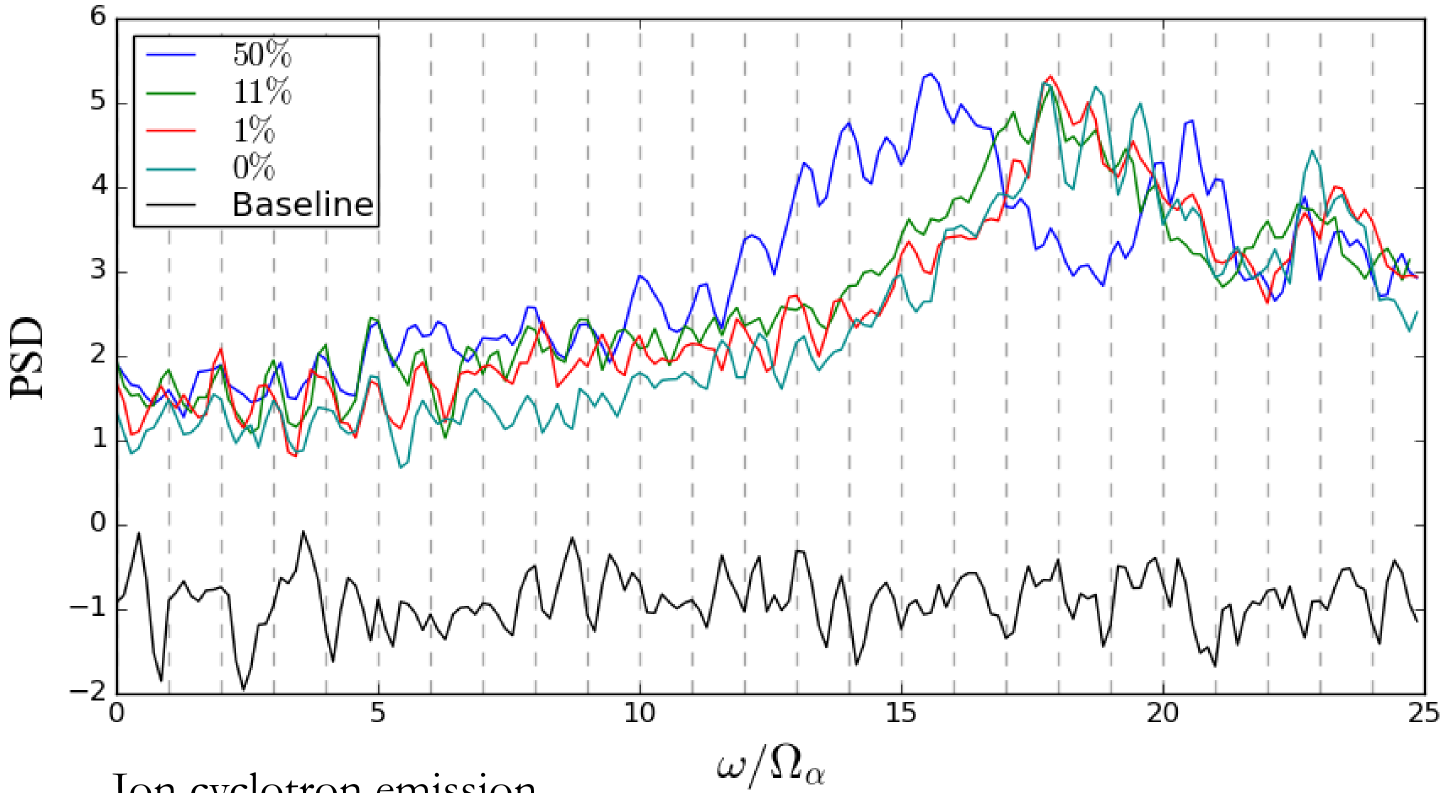
Examples

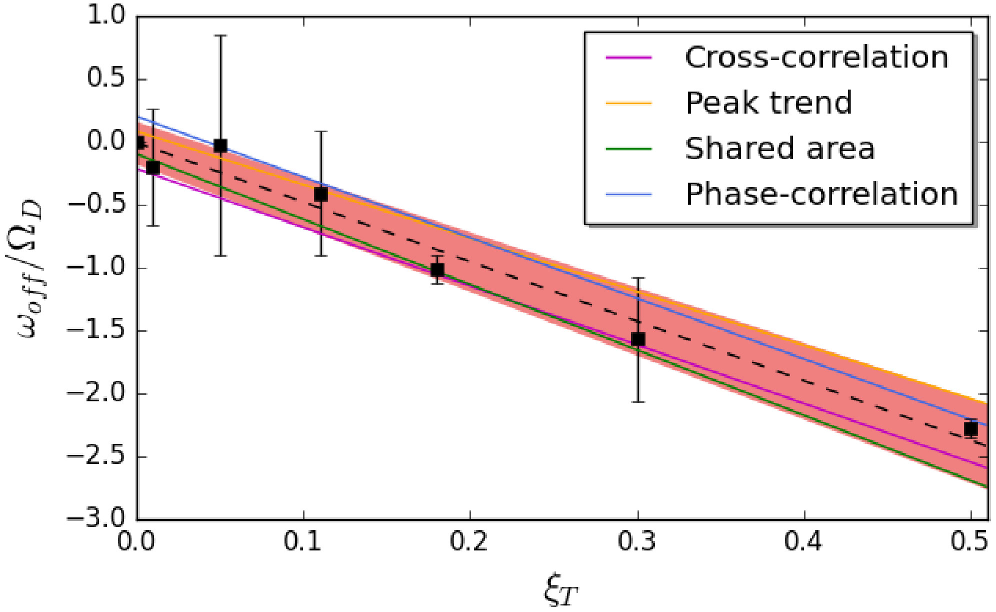
ICE Power spectra

Helioseismology

Financial data

Examples / Power spectra





Helped in revealing power spectral feature trends to further power of ICE as diagnostic

<u>Ion cyclotron emission</u>

- Integer harmonics of fusion-born ion cyclotron frequency $n\Omega_\alpha$
- Deuterium-Tritium (DT) plasma, increasing tritium concentration (ξ_T)
- Spectra shifts, use multiple methods to find shift (<u>Motivation</u>)

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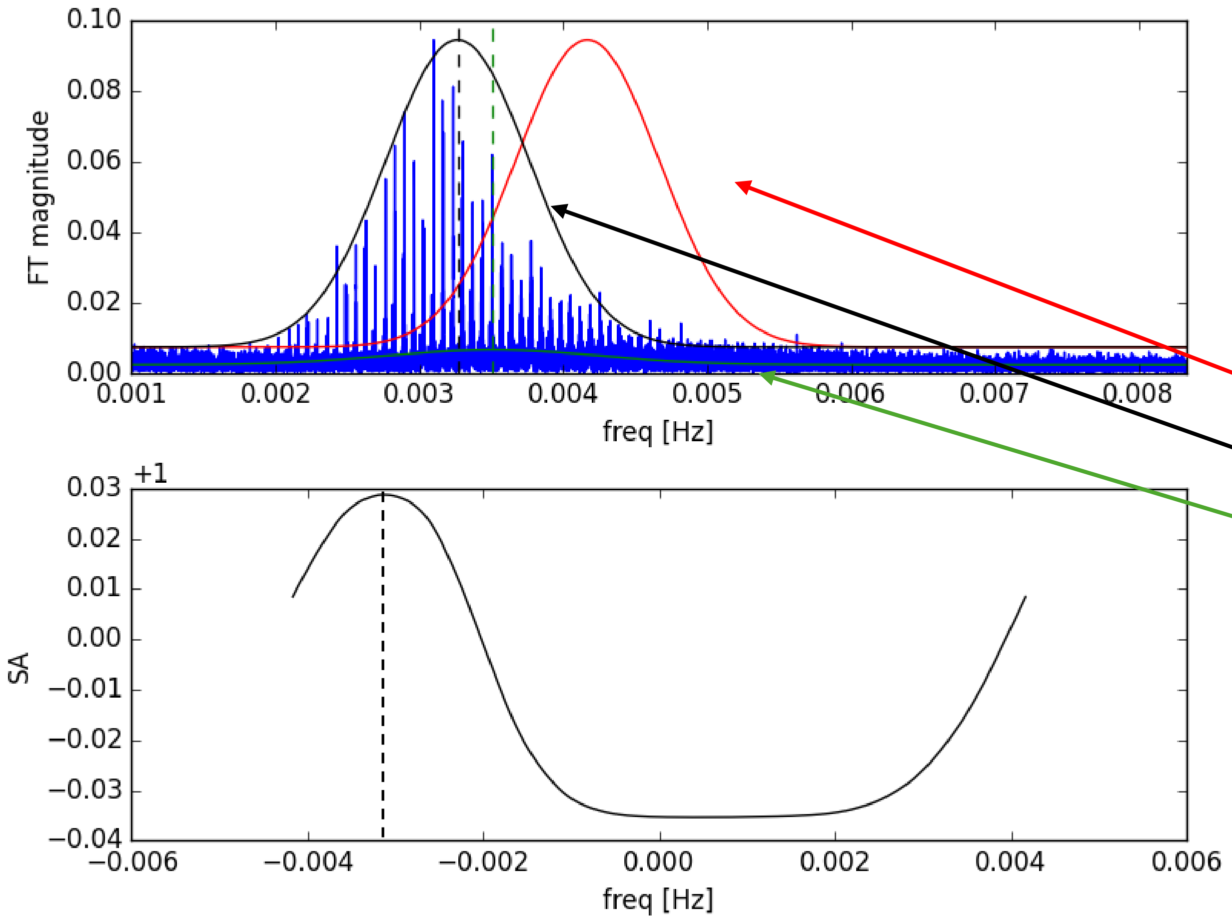
Examples / Helioseismology



Helps find mass and radius of stars

FFT of our Sun's Doppler velocities from the GOLF satellite (total time = 30 days)

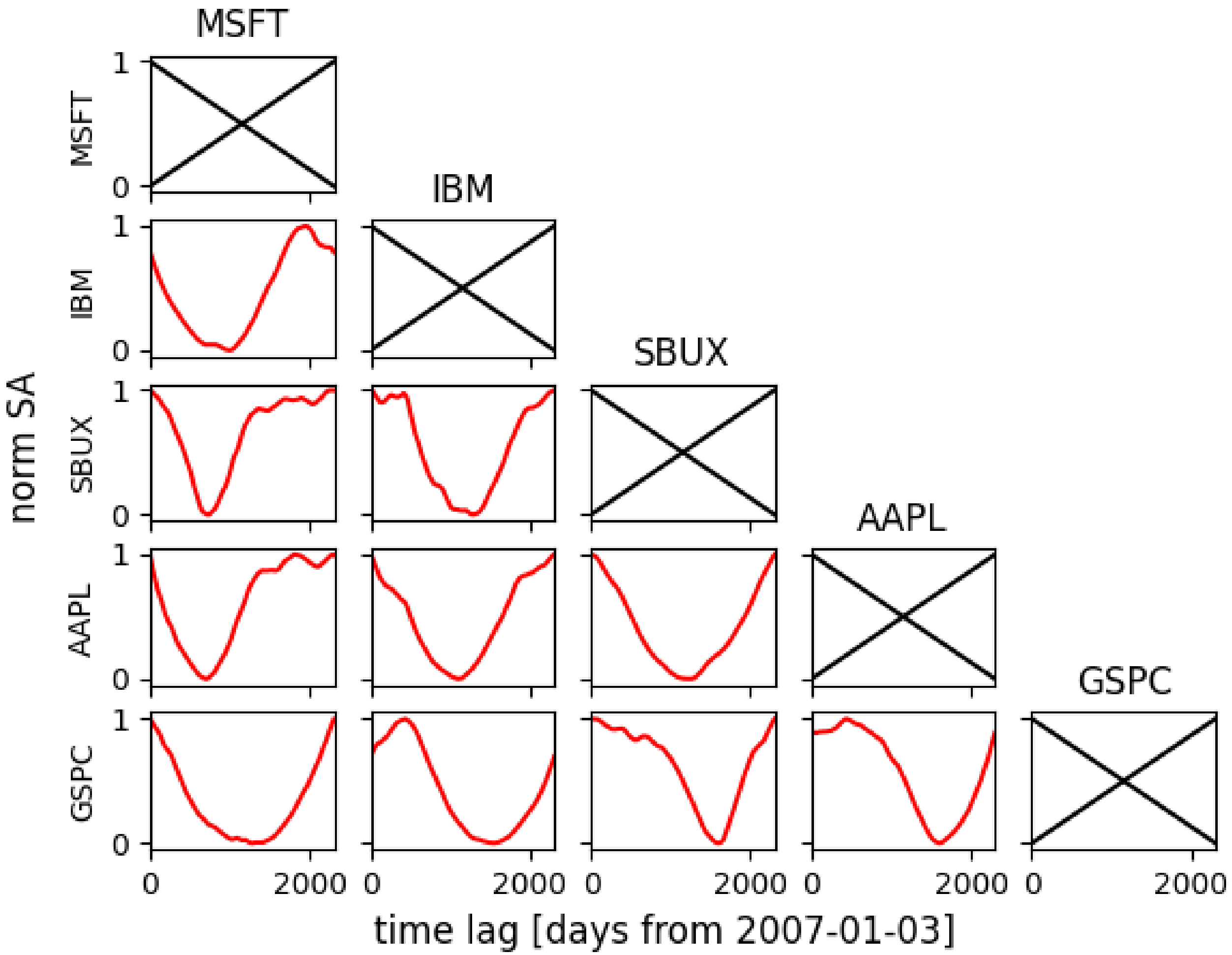
Example Gaussian curve (**red**)
Shared area curve (**black**) $f_{max} = 3265\mu\text{Hz}$
Fitted exponential to *all* data (**green**) $f_{max} = 3509\mu\text{Hz}$

Actual $f_{max} = 3261 \pm 4$ (μHz)*

Room to improve; vis-à-vis shared area model curve

*Howe, R. et al. 2020. *Solar cycle variation of vmax in helioseismic data and its implications for asteroseismology*. MNRAS, 493(1), pp.L49-L53.

Examples / Financial data



- Data from 1st March 2007 to 1st March 2016
- Auto SA removed ($\text{stock}^{(0,0)} \times \text{stock}^{(0,0)}$)
- Row (i) represents sliding SA curve over column (j), reflected plots would reveal inverse time lags
- Normalised SA between 0 and 1 (measure of "correlation" between stock prices)

Thank you for listening